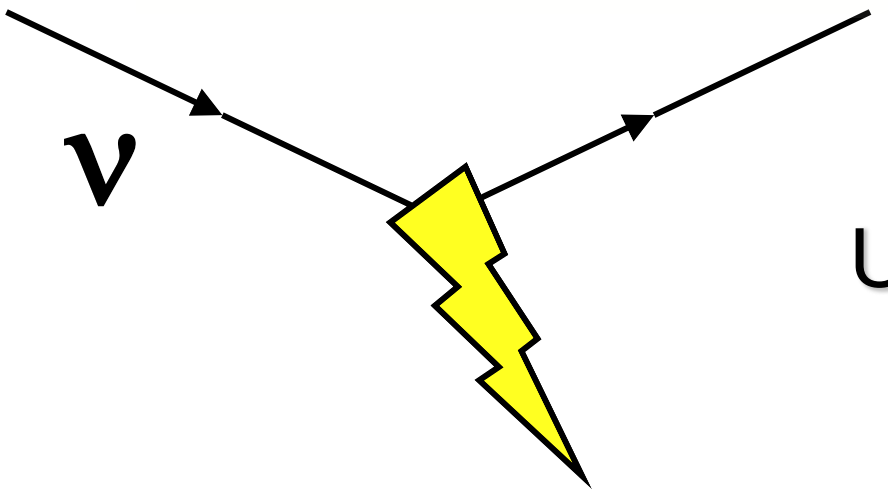
How Neutrinos Interact



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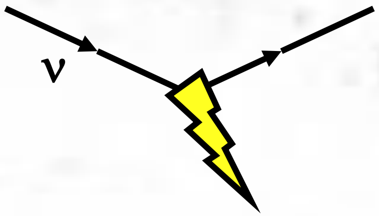
1 July 2010

Outline

- How we measure interactions
  - What is a “cross-section”?
- Weak interactions and Neutrinos
  - Point-like scattering made simple
- Complications
  - Masses, not so point-like scattering, inelastic processes

Cross-Sections

What is a Cross-Section?

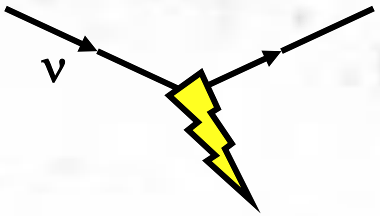


- Define a cross-section so that

$$N_{\text{events}} = \Phi \times \sigma$$

  - where Φ is flux, particles per area
  - and σ is the cross-section
- Example: throw darts at balloons (radius *r*) in a cube (side *L*)
  - What is Φ for a single dart? What is *N*?
  - What, therefore, is the cross-section for one balloon? *n* balloons (for large *L*)?

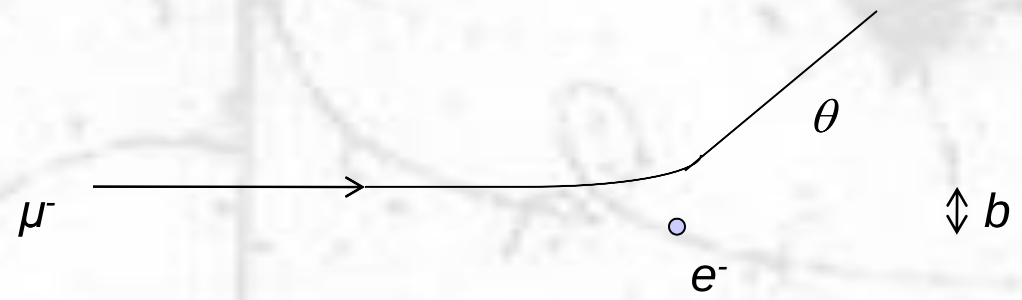
A “Real” Cross-section



- How do two protons interact?
  - Strong interaction is strong inside a proton, but not strong outside a proton
  - Size of proton is 0.87×10^{-15}m (charge radius)
- Calculate a geometric cross-section
  - At “high” (relativistic) energies, the answer should be 4×10^{-30}m^2 (a “barn”=10^{-28}m^2)
  - Geometric answer for proton-proton is close
    (at least if we did it correctly!)

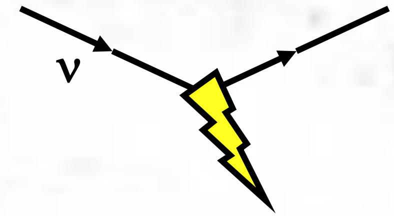
A Different Cross-Section

- Consider $\mu^- e^- \to \mu^- e^-$
  - Can we use the same approach? What is the "size" of an electron or muon?
  - As far as we know, down to $\sim 10^{-19}$m, they have no size. Then how do they interact?



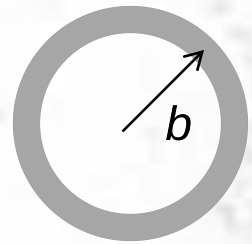
  - Electric field scatters by an angle, θ, related to the distance of closest approach ("impact parameter"), b

A Different Cross-Section *(cont'd)*



- Inverse square law & electron charge $\rightarrow b \propto \dfrac{e^2}{E_\mu} \cot^2 \theta/2$
- What is σ in this case?

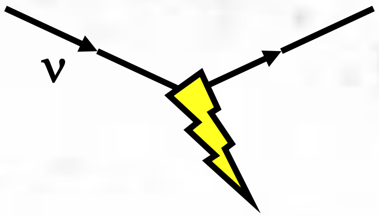
$$d\sigma = 2\pi b\, db$$



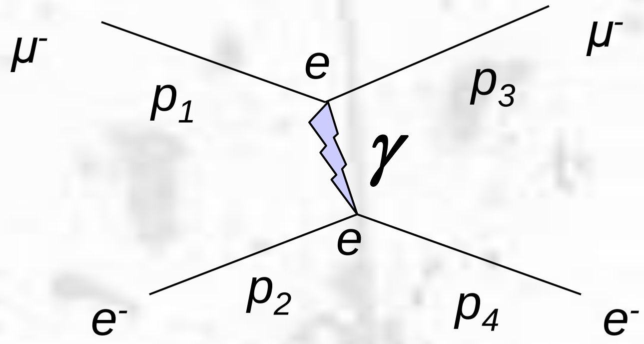
- No limit! But in a practical experiment, only care about b up to some minimum θ. $d\sigma/d\theta$ is always defined.

$$\frac{d\sigma}{d\Omega}(\theta) \propto \frac{e^4}{E_\mu^2} \frac{1}{\sin^4 \theta/2}$$

A Different Cross-Section *(cont'd)*



- Tedious algebra allows us to recast this…



$$s \equiv \left(p_1 + p_2 \right)^2$$

s is total center-of-mass energy squared

$$t \equiv \left(p_1 - p_3 \right)^2$$

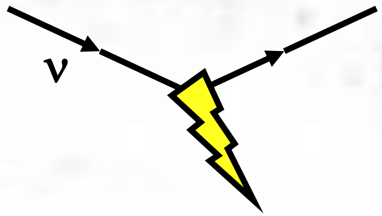
$$\equiv -Q^2 \approx 4E_\mu^{initial} E_\mu^{final} \sin^2 \theta/2$$

$$\frac{d\sigma}{dQ^2} = \frac{1}{64s} \frac{1}{\left| p_1^{cm} \right|^2} \left| \mathfrak{M} \right|^2$$

"matrix element" for process

$$\mathfrak{M} \propto \frac{e^2}{Q^2 + M_\gamma^2} = \frac{e^2}{Q^2}$$

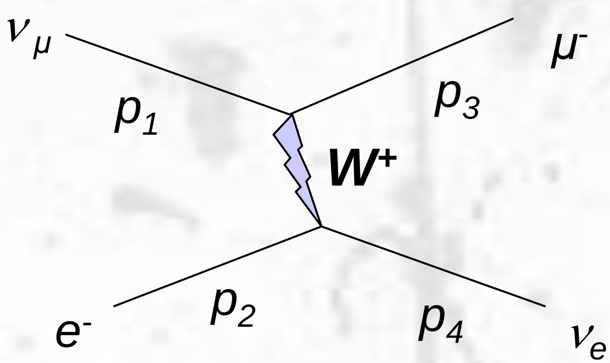
$1/r^2$ field gives $1/Q^4$ cross-section



Weak Interactions

A Weak Cross-Section

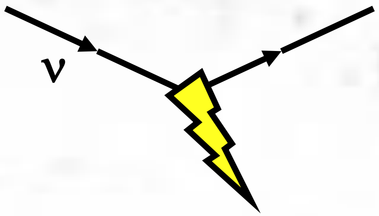
- Now let's look at a weak process involving neutrinos, which only feel weak interactions



$$
\begin{aligned}
s &\equiv \left(p_1 + p_2\right)^2 \\
&= \left(E_\nu + m_e\right)^2 - \left(\vec{p}_\nu\right)^2 \\
&= E_\nu^2 - p_\nu^2 + m_e^2 + 2E_\nu m_e \approx 2E_\nu m_e
\end{aligned}
$$

- For a realistic experiment, the neutrino beam energy can't be much over 100 GeV, so the total center of mass energy is less than 1 GeV
- But W boson rest mass is 80 GeV!!

A Weak Cross-Section *(cont'd)*



- Solution… "borrow" energy from the vacuum for a short time

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

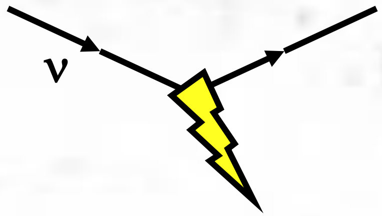
$$\therefore t \sim \hbar / \Delta E$$

- Numerically, if we have to borrow 80 GeV, t~8x10^{-27}s
- Implies the W can travel only 2.5x10^{-18}m, so the weak interaction is very short range

$$\mathfrak{M} \propto \frac{1}{Q^2 + M_W^2} \approx \text{constant}$$

short range field gives momentum-transfer independent matrix element

Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

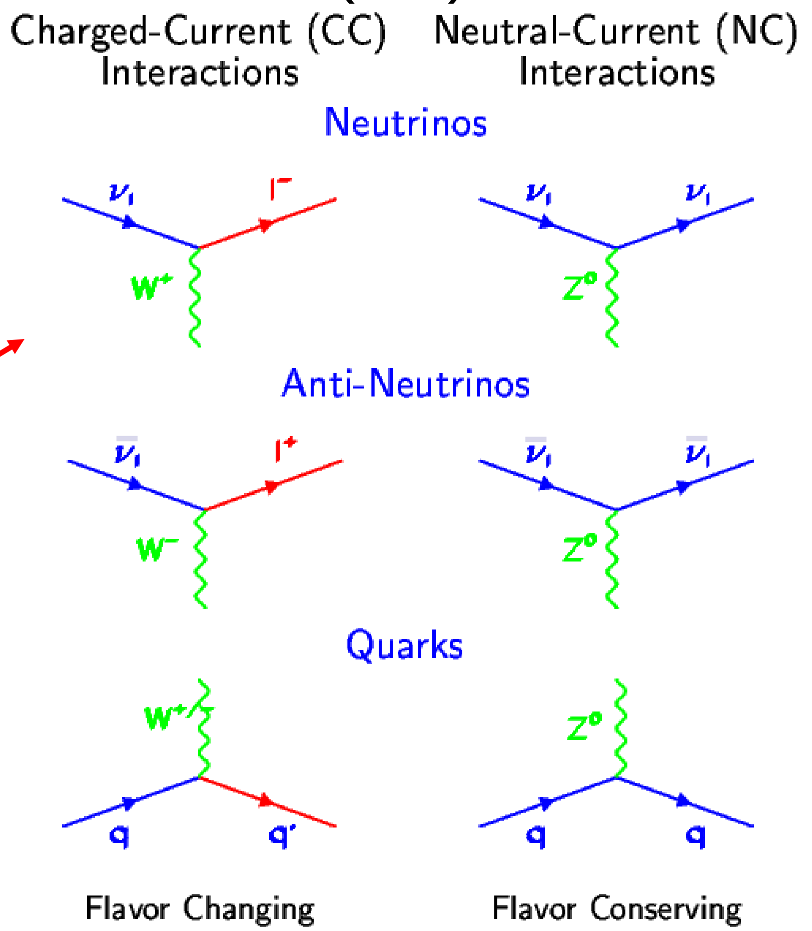
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

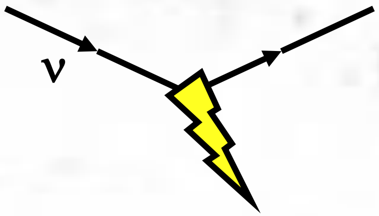
Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

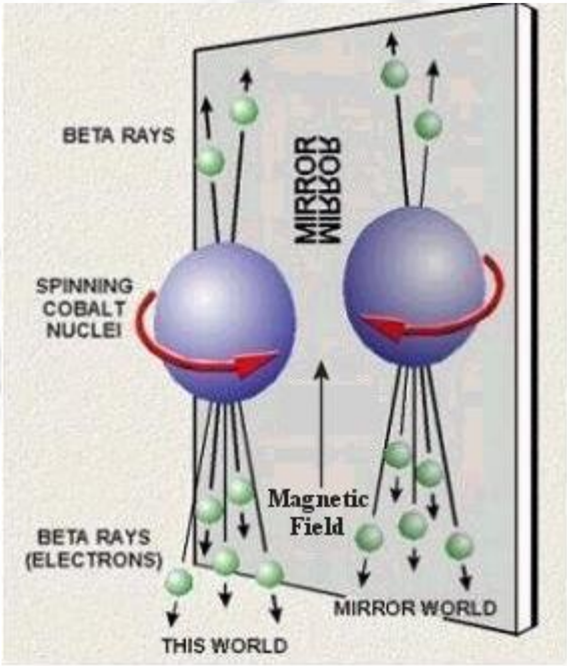
$$l^+ \Rightarrow \bar{\nu}_l$$



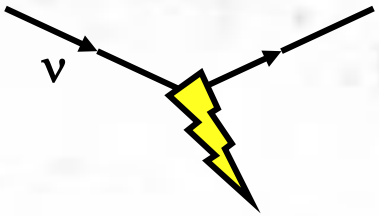
Parity Violation



- Nature is not symmetric in the mirror!
- The weak interaction maximally violates parity
  - Charged weak interaction only affects left-handed states (for a massless particle, that means spin and direction are opposite)
  - First observed in beta decay of nuclei with spin
  - *This was an enormous surprise when first observed… a classic case of experimental evidence being accessible for long before it was puzzled out.*



Electroweak Theory

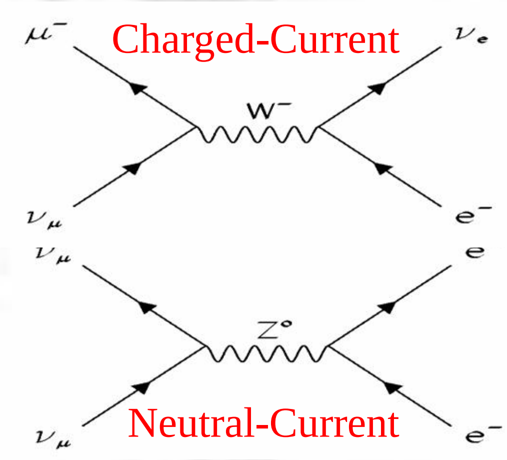


- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM

    $\Rightarrow$ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings, $g' = g \tan\theta_W$

Z Couplings	g_L	g_R
ν_e, ν_μ, ν_τ	1/2	0
e, μ, τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
d, s, b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

$$e = g\sin\theta_W,\; G_F = \frac{g^2\sqrt{2}}{8M_W^2},\; \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - Right-handed neutrino has ***NO*** interactions!

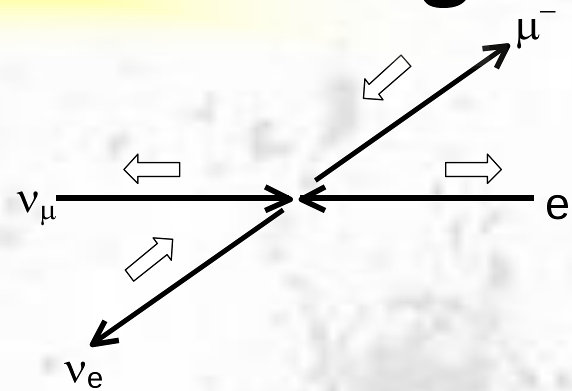
Neutrino-Electron Scattering

- Inverse μ–decay:

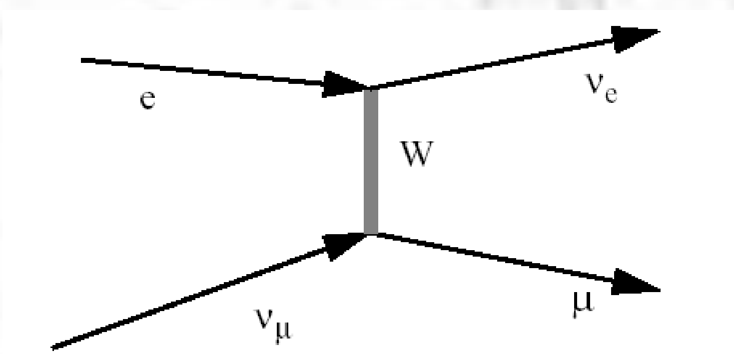
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)

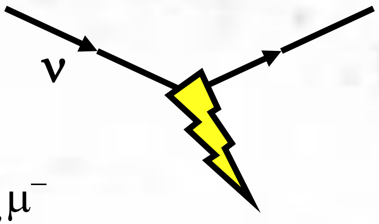


$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} \frac{d\sigma}{dQ^2} dQ^2$$



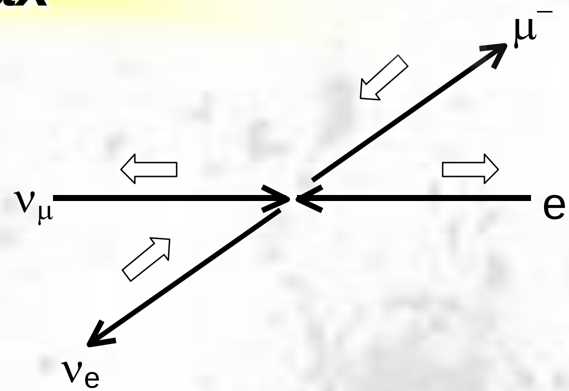
$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max}}{M_W{}^4}$$



What is Q^2_{max}?

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

*Let's work in the center-of-mass frame. Assume, **for now**, we can neglect the masses*

$$\underline{e} \approx (E_\nu^*, 0, 0, -E_\nu^*)$$

$$\underline{\nu}_e \approx (E_\nu^*, -E_\nu^* \sin\theta^*, 0, -E_\nu^* \cos\theta^*)$$

$$Q^2 = -\left(\underline{e}^2 + \underline{\nu}_e{}^2 - 2\underline{e}\bullet\underline{\nu}_e\right)^2$$

$$\approx -\left[-2E_\nu^{*2}\left(1 - \cos\theta^*\right)\right]$$

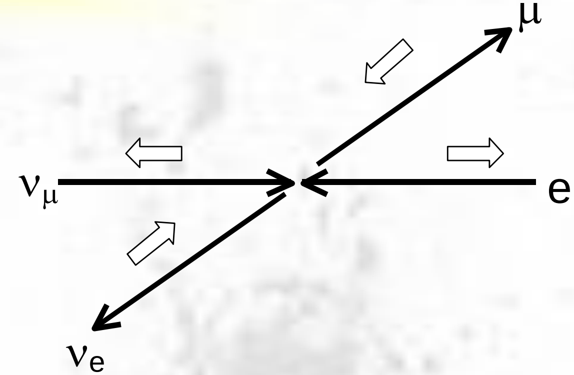
$$0 < Q^2 < \left(2E_\nu^*\right)^2 \approx \left(\underline{e} + \underline{\nu}_\mu\right)^2$$

$$0 < Q^2 < s$$

Neutrino-Electron (cont'd)

$$\sigma_{TOT} \propto Q^2_{\max} = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$
$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$

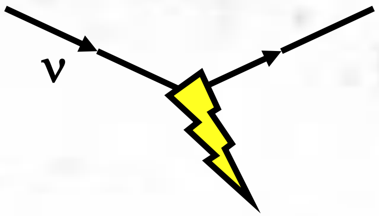


- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)

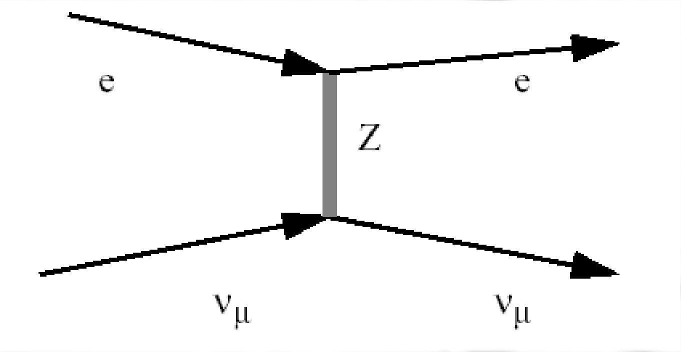
Neutrino-Electron (cont'd)



- Elastic scattering:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

  - Coupling to left *or* right-handed electron
  - Total spin, J=0,1



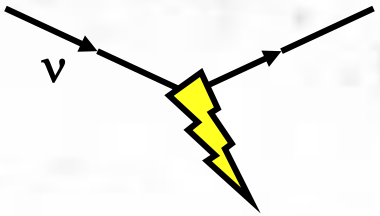
- Electron-Z^0 coupling
  - (LH, V-A): $-1/2 + \sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

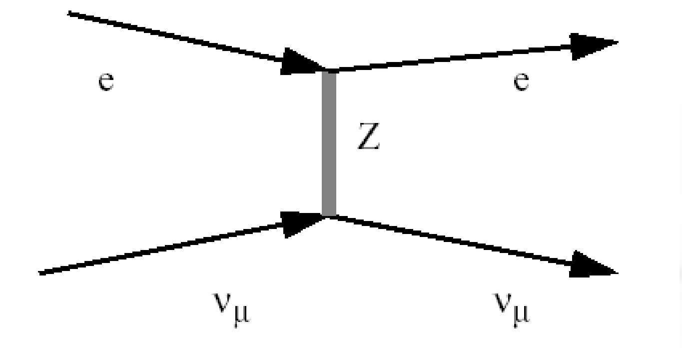
  - (RH, V+A): $\sin^2\theta_W$

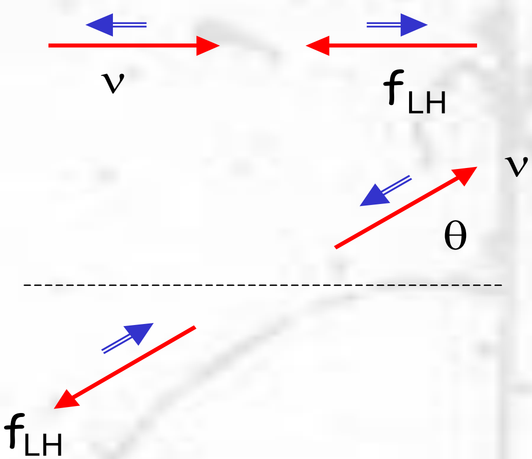
$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

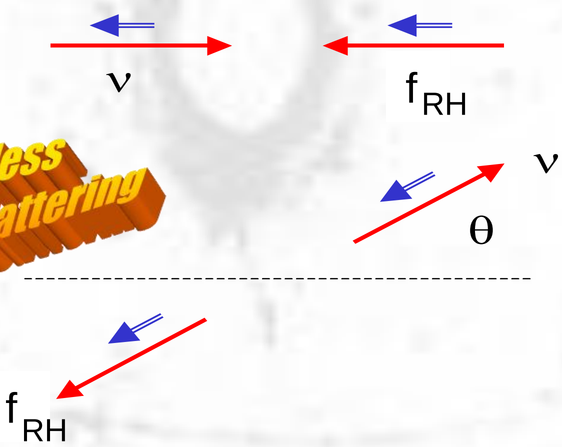
Neutrino-Electron (cont'd)



- What are relative contributions of left *and* right-handed scattering from electron?



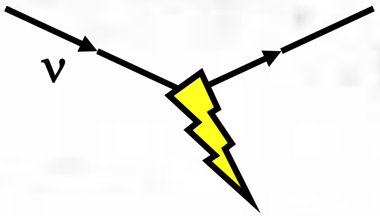




$$\frac{d\sigma}{d\cos\theta} = \mathrm{const}$$

$$\frac{d\sigma}{d\cos\theta} = \mathrm{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

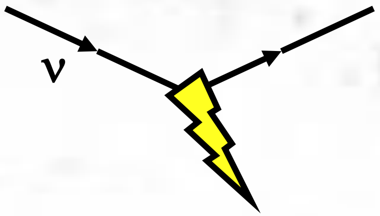
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1 - \cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Flavors and ve Scattering

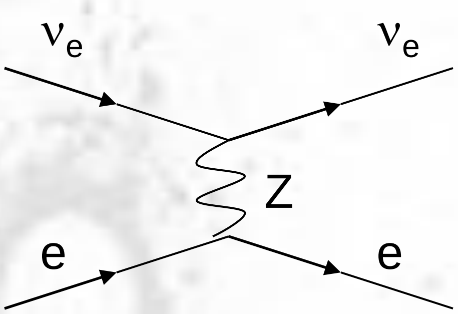


The reaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

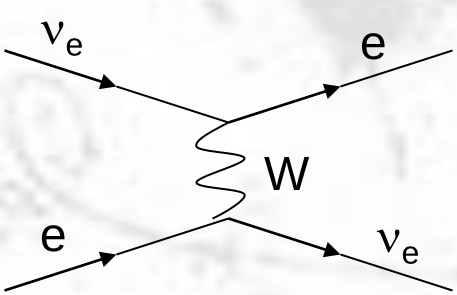
$$\nu_e + e^- \rightarrow \nu_e + e^-$$



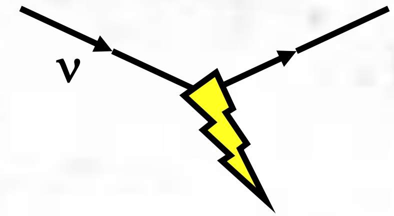
Why?

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

has a second contributing reaction, charged current



Flavors and νe Scattering



Show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{\text{LH}} \right|^2$$

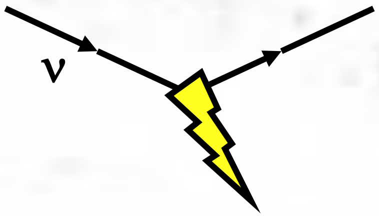
For electron…	LH coupling	RH coupling
Weak NC	$-1/2+\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

)

We have to show the interference between CC and NC is constructive.

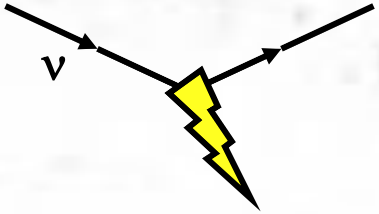
The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is -1/2. We add the associated amplitudes… and get $-1+\sin^2\theta_W \approx -3/4$



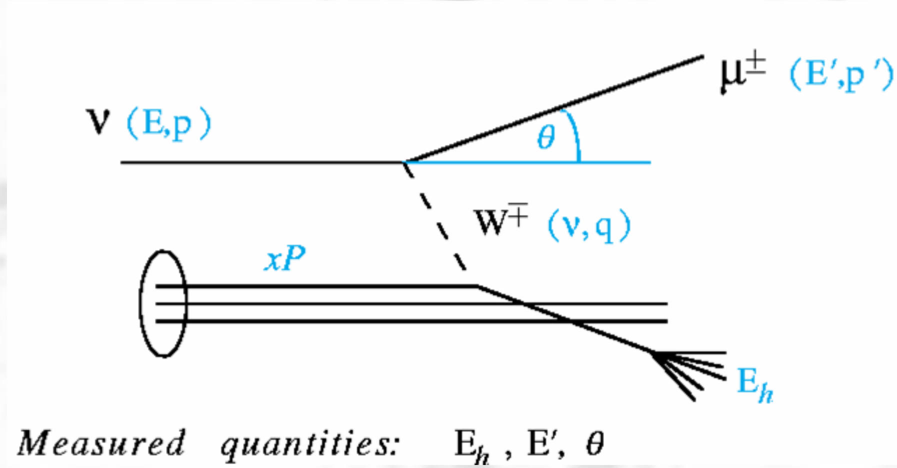
Neutrino-Nucleon Scattering

Scattering Variables



Scattering variables given in terms of invariants

- More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



4-momentum Transfer^2: $Q^2 = -q^2 = -\left(p' - p\right)^2 \approx \left(4EE' \sin^2(\theta/2)\right)_{Lab}$

Energy Transfer: $\nu = \left(q\cdot P\right)/M_T = \left(E - E'\right)_{Lab} = \left(E_h - M_T\right)_{Lab}$

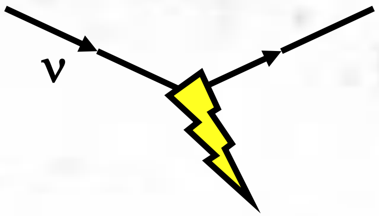
Inelasticity: $y = \left(q\cdot P\right)/\left(p\cdot P\right) = \left(E_h - M_T\right)/\left(E_h + E'\right)_{Lab}$

Fractional Momentum of Struck Quark: $x = -q^2/2\left(p\cdot q\right) = Q^2/2M_T\nu$

Recoil Mass^2 : $W^2 = (q+P)^2 = M_T{}^2 + 2M_T\nu - Q^2$

CM Energy^2 : $s = (p+P)^2 = M_T{}^2 + \dfrac{Q^2}{xy}$

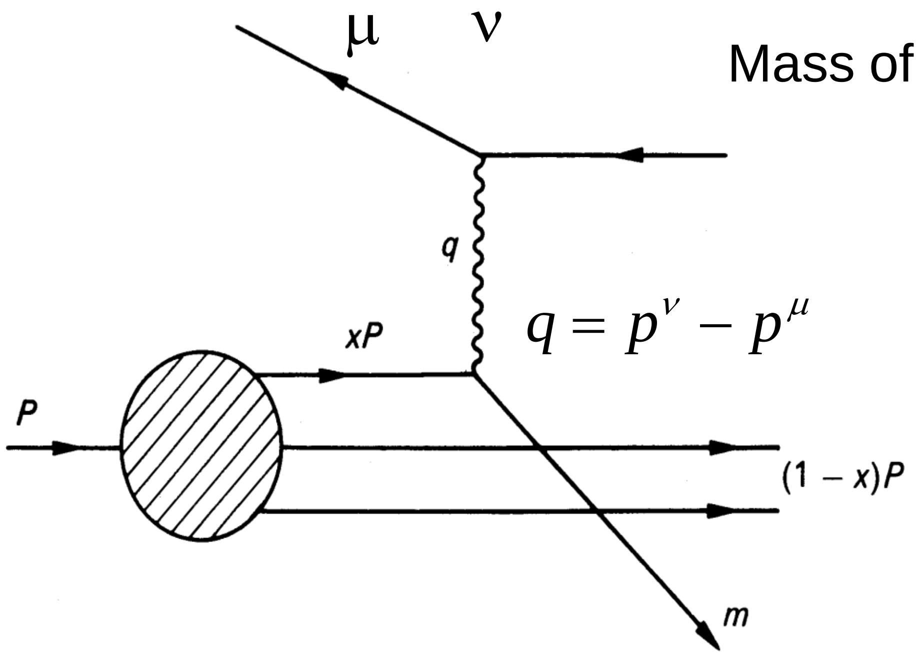
Parton Interpretation



Mass of target quark $$m_q^{\;2} = x^2 P^2 = x^2 M_T^2$$

Mass of final state quark

$$m_{q'}^{\;2} = (xP + q)^2$$



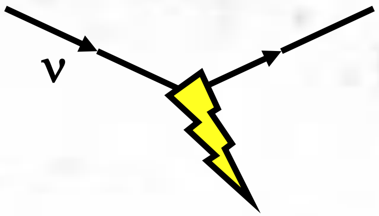
$$q = p^{\nu} - p^{\mu}$$

In "infinite momentum frame", x is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

Neutrino scatters off a point-like parton inside the nucleon.

Valid picture at high energies



Why is cross-section so large?

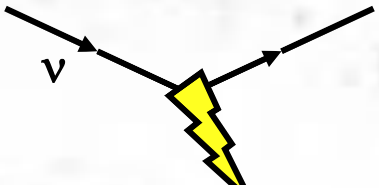
- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

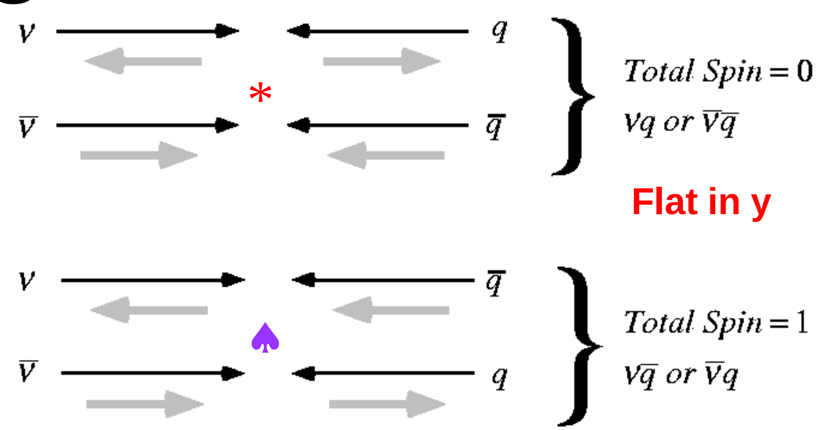
$$s = m_e^{\ 2} + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger σ_{TOT}

Chirality, Charge in CC ν-q Scattering



- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



Flat in y

$1/4(1+\cos\theta^*)^2 = (1-y)^2$

$\int(1-y)^2 dy = 1/3$

point-like scattering implies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{d}(x) + x\overset{\spadesuit}{\bar{u}}(x)(1-y)^2 \right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{\bar{d}}(x) + x\overset{\spadesuit}{u}(x)(1-y)^2 \right)$$

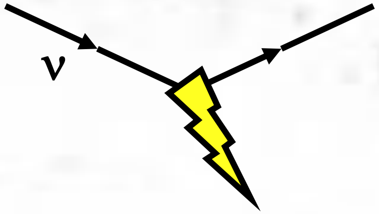
but what is this "q(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

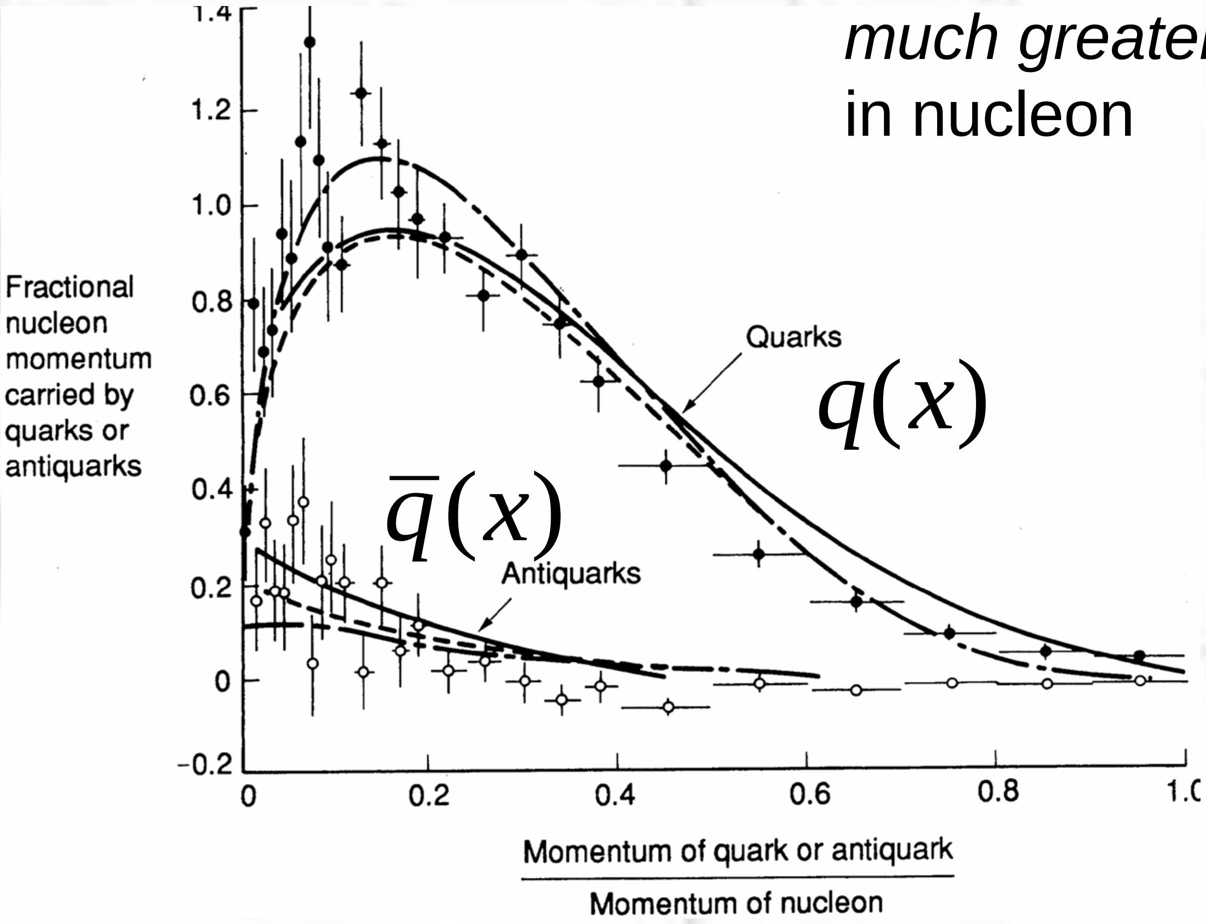
$$\nu d \rightarrow \mu^- u$$

$$\bar{\nu} u \rightarrow \mu^+ d$$

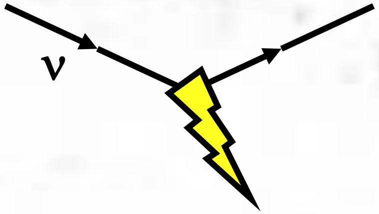
Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon



y distribution in Neutrino CC DIS



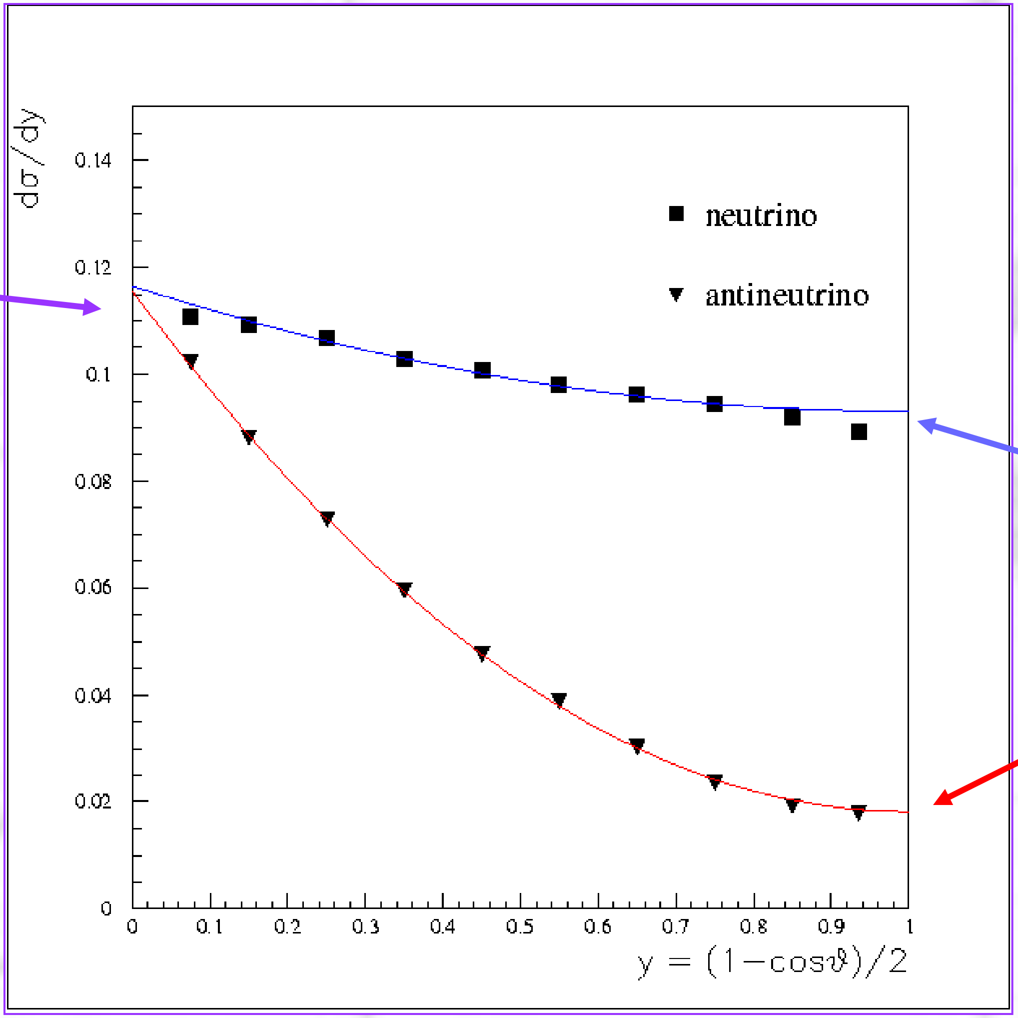
$$\frac{d\sigma(\nu q)}{dxdy} = \frac{d\sigma(\bar{\nu}\bar{q})}{dxdy} \propto 1$$

$$\frac{d\sigma(\nu \bar{q})}{dxdy} = \frac{d\sigma(\bar{\nu} q)}{dxdy} \propto (1-y)^2$$

y=0:

Quarks & anti-quarks

Neutrino and anti-neutrino identical

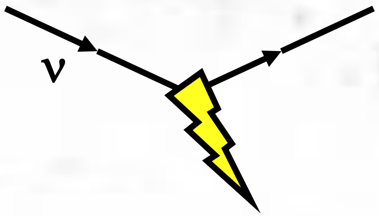


y=1:

Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

$$\sigma^{\bar{\nu}} \approx \tfrac{1}{2}\sigma^{\nu}$$

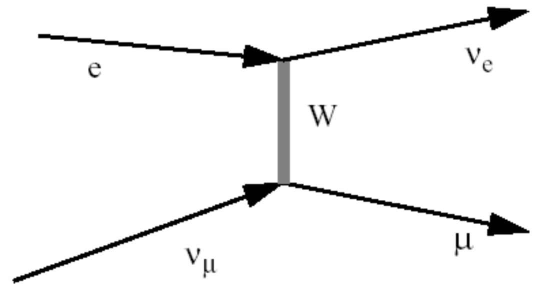


Complications

Lepton Mass Effects

- Let's return to Inverse μ–decay:

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



  - What changes in the presence of final state mass?
    - pure CC so always left-handed
    - BUT there must be finite Q^2 to create muon in final state!

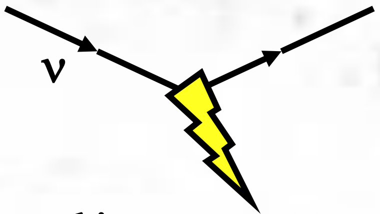
$$Q^2_{\min} = m^2_\mu$$

$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2} \approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

  - see a suppression scaling with **(mass/CM energy)²**
    - can be generalized…

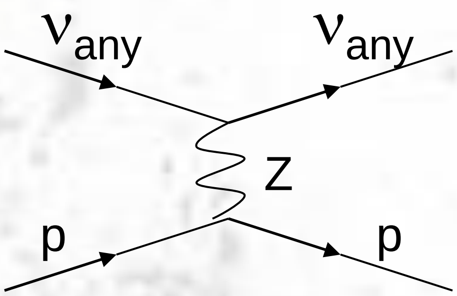
$$\sigma_{TOT} = \frac{G_F^2 (s - m_\mu^2)}{\pi} = \left[\sigma_{TOT}{}^{\text{(massless)}}\right]\left(1 - \frac{m_\mu^2}{s}\right)$$

What about other targets?



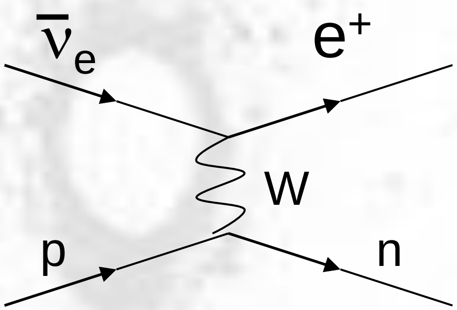
- Imagine now a proton target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$



  - "Inverse beta-decay" (IBD):

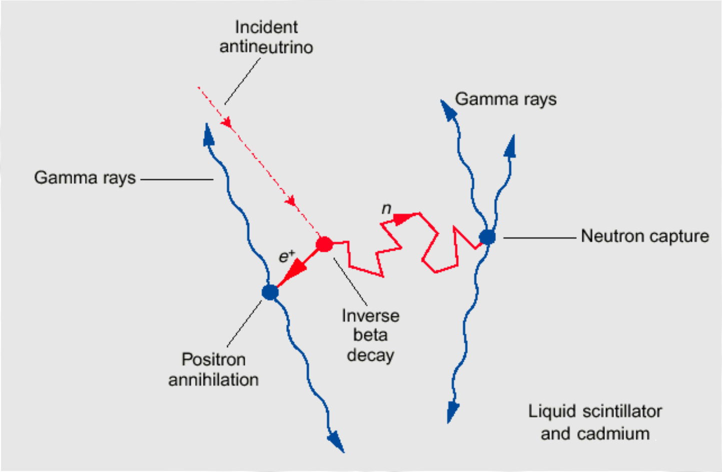
$$\bar{\nu}_e + p \rightarrow e^+ + n$$



  - and its close cousin:

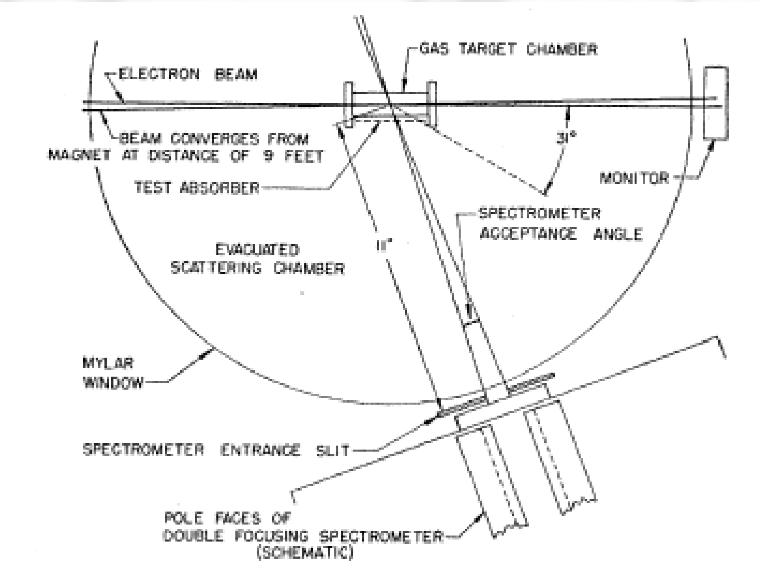
$$\nu_e + n \rightarrow e^- + p$$

  - IBD was the Reines and Cowan discovery signal for the neutrino

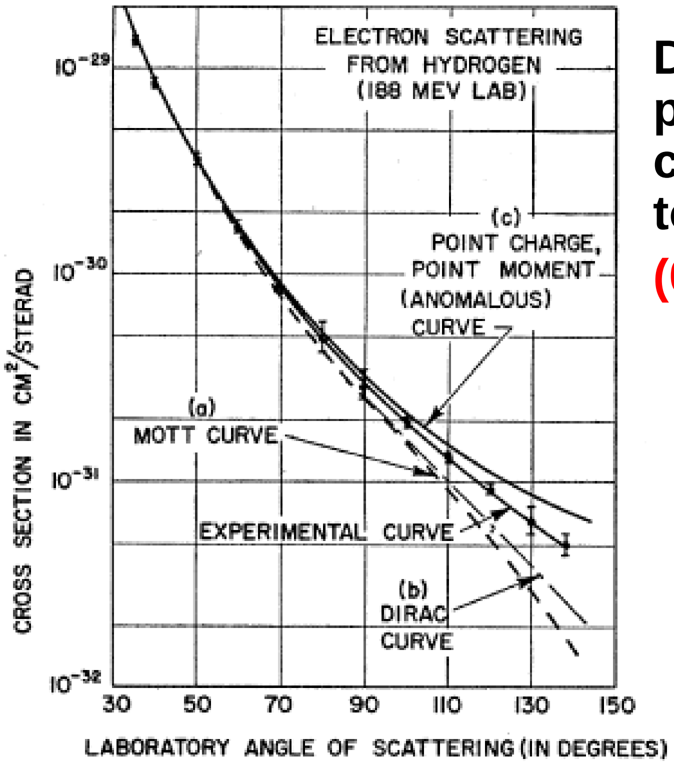


Proton Structure

- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 1$
  - “form factors” related to finite size



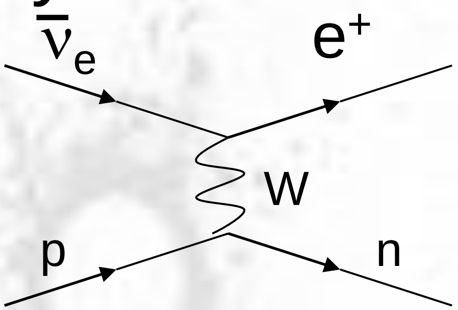
McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford



Determined proton RMS charge radius to be
(0.7 0.2) x10^{-13} cm

Final State Mass Effects

- In IBD, $\bar{\nu}_e + p \to e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



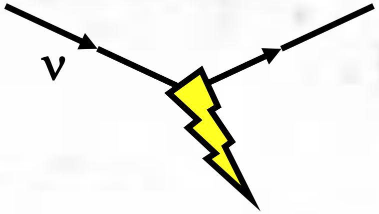
- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\min} \approx \dfrac{\left(M_n + m_e\right)^2 - M_p^{\,2}}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)



- Define δE as E_ν-E_ν^{min}, then

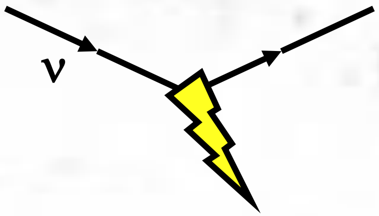
$$s_{\text{initial}} = M_p^2 + 2M_p\left(\delta E + E_\nu^{\text{min}}\right)$$
$$= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2$$
$$= 2\delta E \times M_p + \left(M_n + m_e\right)^2$$

- Remember the suppression generally goes as

$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}^{\;2}}{s} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p^{\;2}} \dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$

Putting it all together…

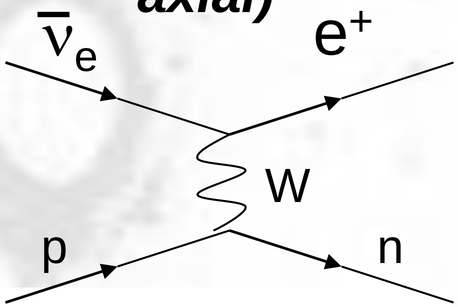


$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3 g_A{}^2\right)$$

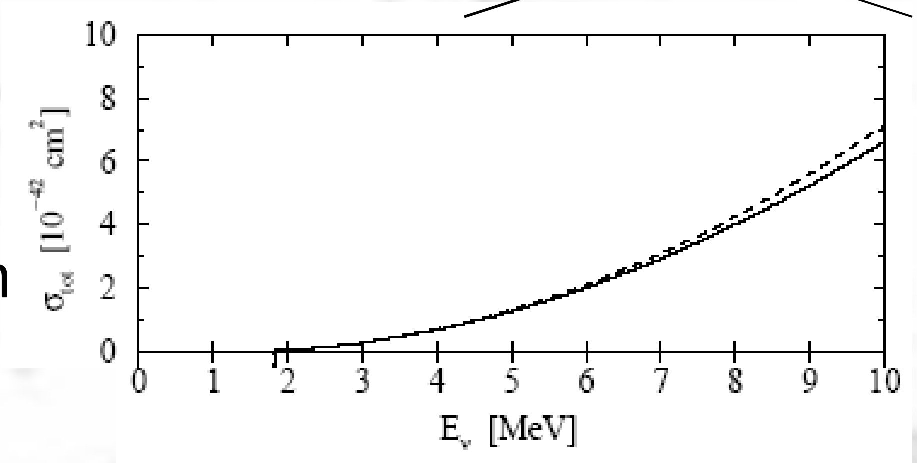
quark mixing!

final state mass suppression

proton form factors (vector, axial)

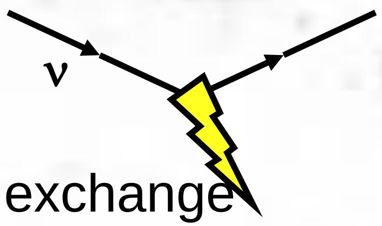


- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively) g_V, $g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n

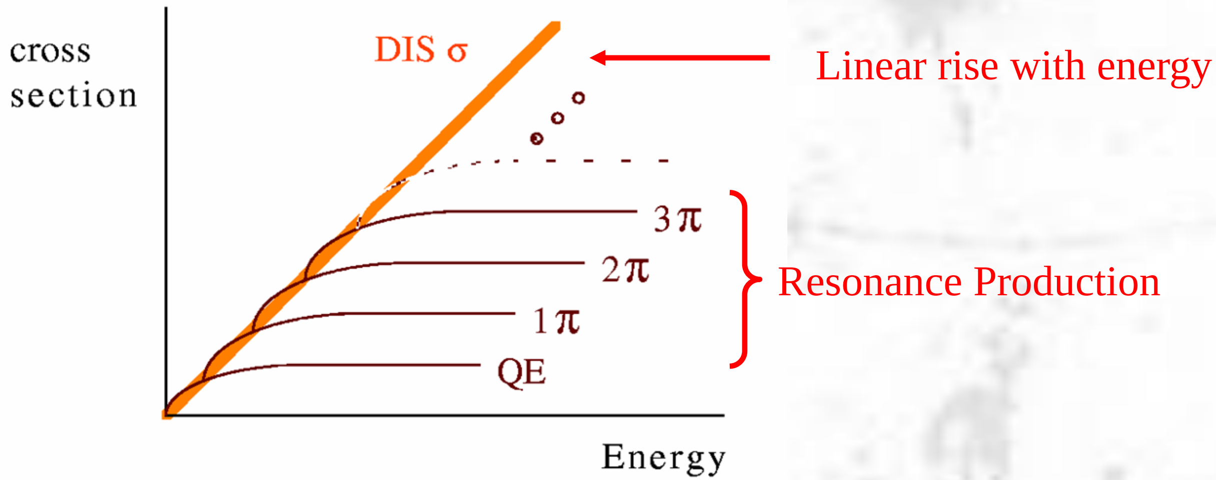


GeV Cross-Sections

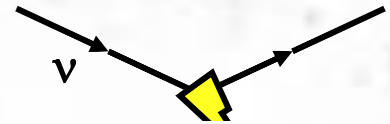
Neutrino-Nucleon 'n a Nutshell

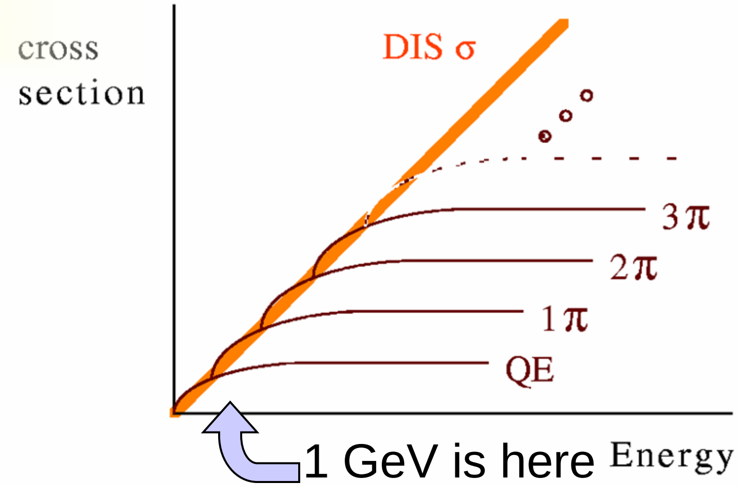


- Charged - Current: $W^{\pm}$ exchange
  - Quasi-elastic Scattering:
    (Target changes but no break up)
    $\nu_\mu + n \to \mu^- + p$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \to \mu^- + p + \pi^0$ $(N^*$ or $\Delta)$
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \to \mu^- + \text{quark}'$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \to \nu_\mu + N$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \to \nu_\mu + N + \pi$ $(N^*$ or $\Delta)$
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \to \nu_\mu + \text{quark}$

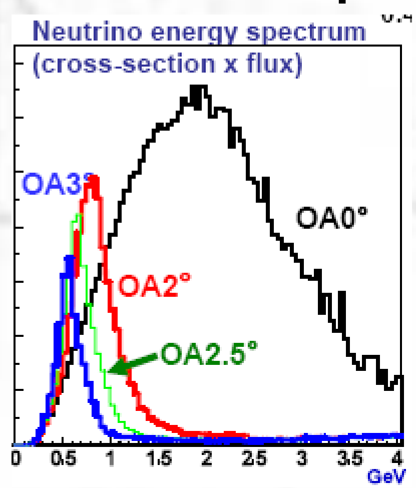


What's special about it? Why do we care?





- Remember this picture?
  - 1-few GeV is exactly where these additional processes are turning on
  - It's not DIS yet! Final states & threshold effects matter
- Why is it important? Example: T2K



Goals:

1. $\nu_\mu \rightarrow \nu_e$
2. ν_μ disappearance

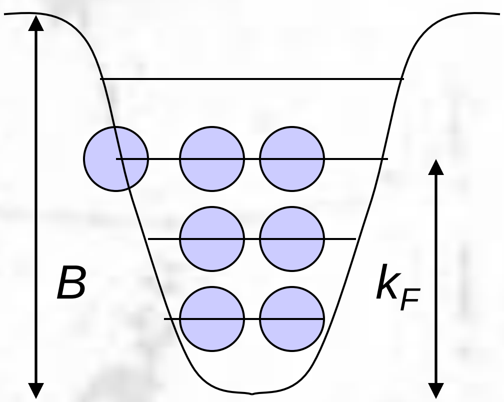
E_ν is 0.4-2.0 GeV

Nuclear Effects in Elastic Scattering

- Two effects
  - In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
    - o Often handled in a “Fermi Gas” model of nucleons filling available states up to some initial state Fermi momentum, k_F

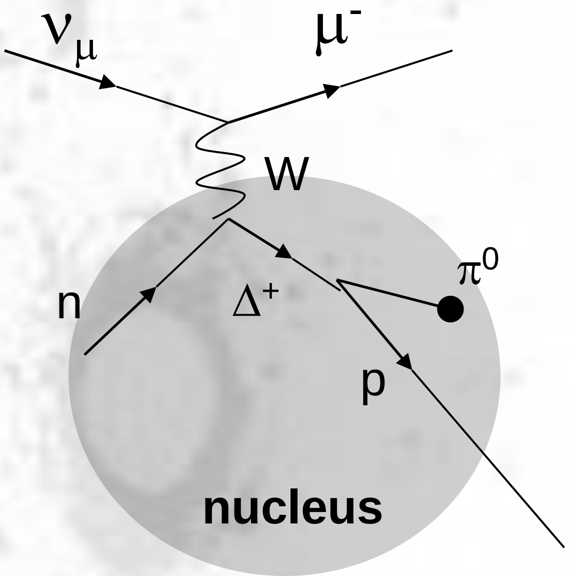
ν

  - Outgoing nucleon can interact with the target
    - o Usually treated as a simple binding energy
    - o Also, Pauli blocking for nucleons not escaping nucleus… states are already filled with identical nucleon



Nuclear Effects in Elastic Scattering (cont'd)

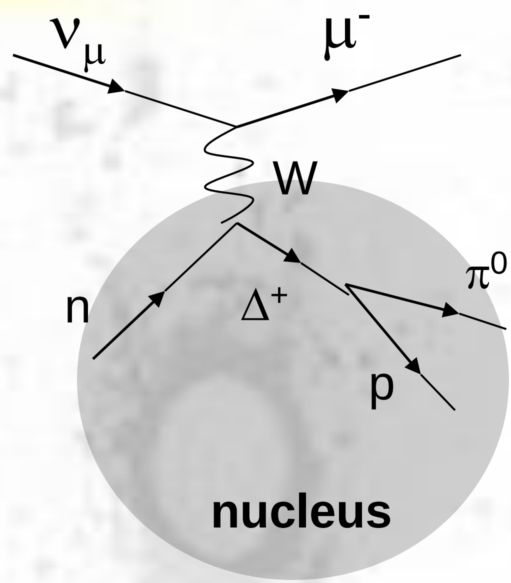
- Also other final states can contribute to apparent "quasi-elastic" scattering through absorption in the nucleus…
  - kinematics may or may not distinguish the reaction from elastic

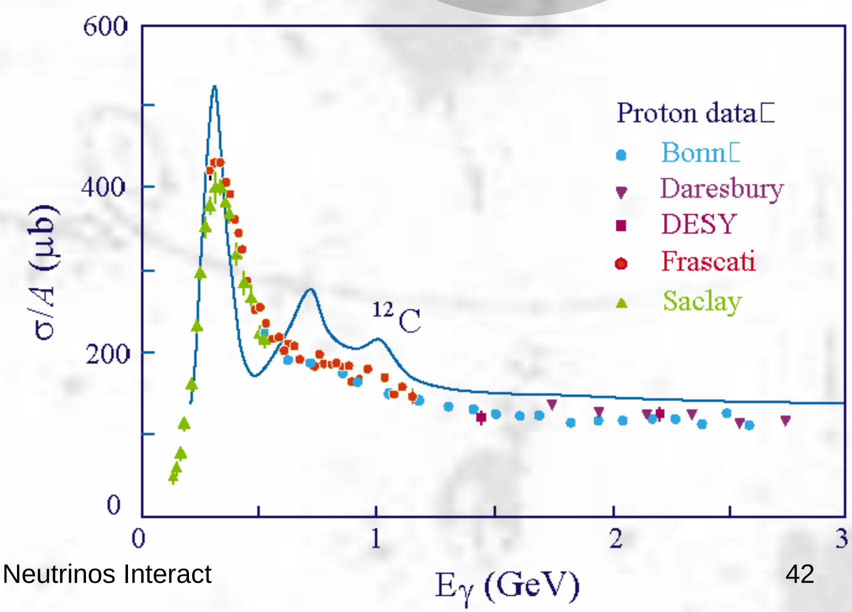


- Theoretical uncertainties are large
  - At least at the 10% level
  - If precise knowledge is needed for target (e.g., water, liquid argon, hydrocarbons), dedicated measurements will be needed
    - Most relevant for low energy experiments, i.e., T2K

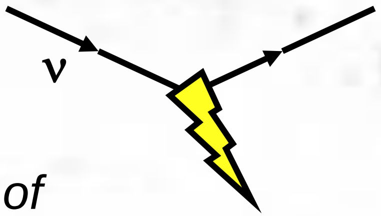
Nuclear Effects in Resonance Region

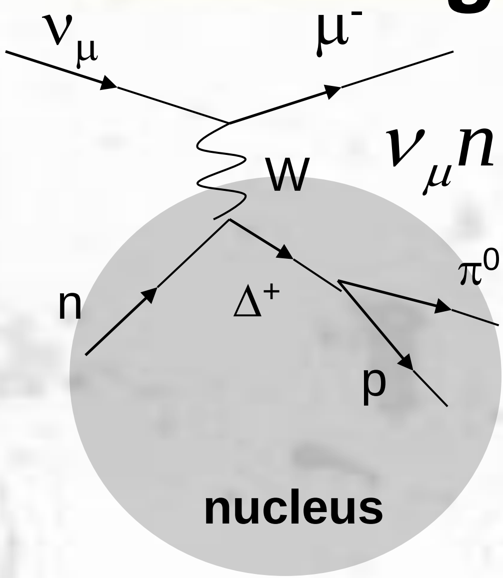
- An important reaction like
$$\nu_\mu n \rightarrow \mu^- p \pi^0$$
(ν_e background) can be modified in a nucleus
- Production kinematics are modified by nuclear medium
  - at right have photoabsorption showing resonance structure
  - line is proton; data is ^{12}C
  - except for first Δ peak, the structure is washed out
  - interactions of resonance inside nucleus





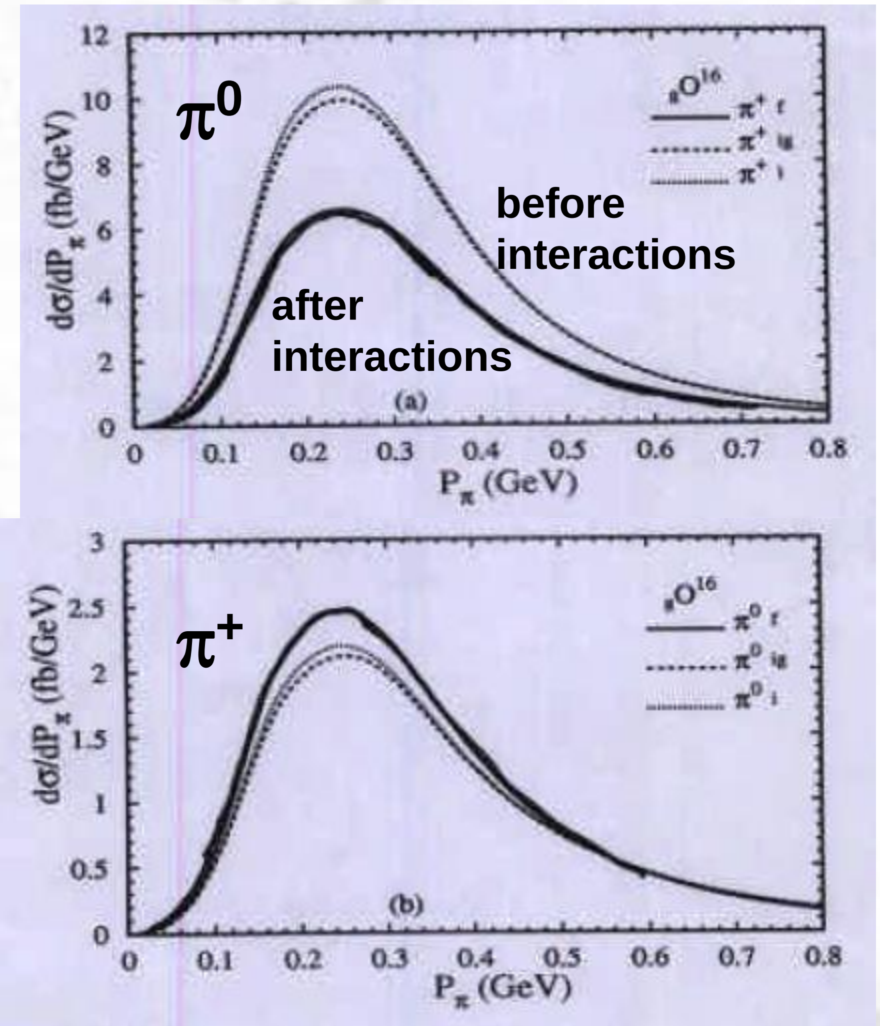
Nuclear Effects in Resonance Region (cont'd)



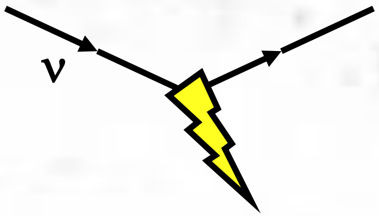


$$\nu_\mu n \rightarrow \mu^- p \pi^0$$

model of E. Paschos, NUINT04

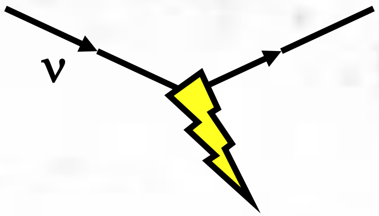


- How does nucleus affect π^0 after production
- Rescattering. Absorption.
- Must measure to predict ν_e backgrounds!



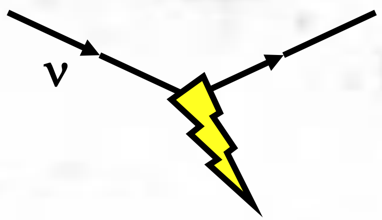
Connections to Low Energy and Ultra-High Energy Cross-Sections

Ultra-High Energies



- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, ANITA)
  - ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
    - o at high Q^2, scaling violations have made most of nucleon momentum carried by sea quarks
    - o see a rise in σ/E_ν from growth of sea at low x
    - o neutrino & anti-neutrino cross-sections nearly equal
  - *Until* Q^2»M_W^2, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$



Where does σ Level Off?

- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- At what beam energy for a target at rest will this happen?

$$Q^2 < s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

$$Q^2 < s_{\text{nucleon}} \approx 2E_\nu m_{\text{nucleon}}$$

$$\frac{M_W^2}{2m_{\text{nucleon}}} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{(80.4)^2\ \text{GeV}^2}{2(.938)\text{GeV}} \sim 3000\text{GeV}$$

Q^2 limit is s.
So won't start to plateau until $s > M_W^2$

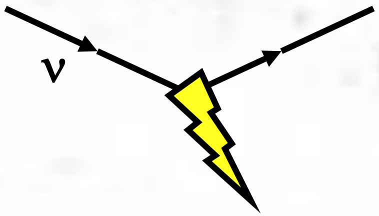
Bonus point realization…

In reality, that is only correct for a parton at x=1. Typical quark x is much less, say ~0.03

$$\frac{M_W^2}{2m_{\text{nucleon}}x} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{3000\text{GeV}}{0.03} \sim 100\text{TeV}$$

Ultra-High Energies



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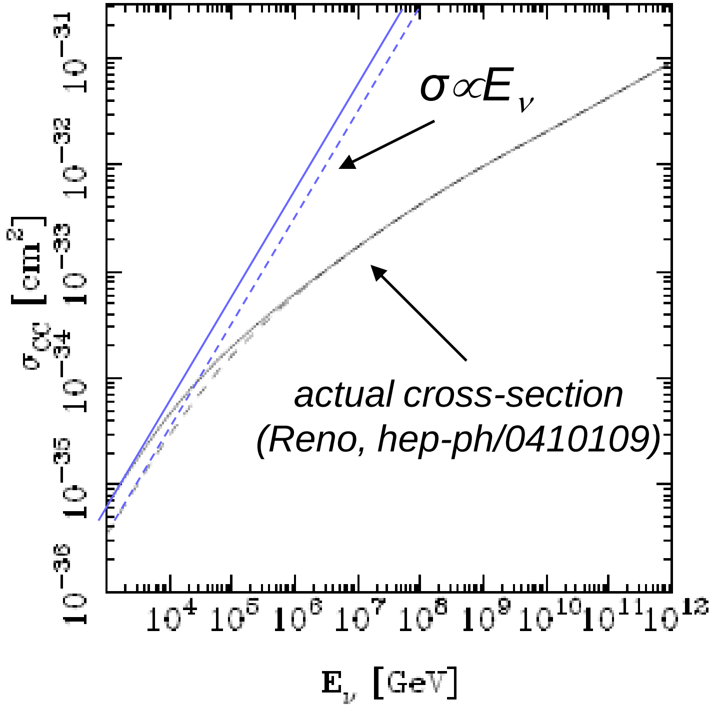
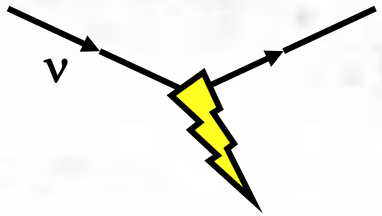
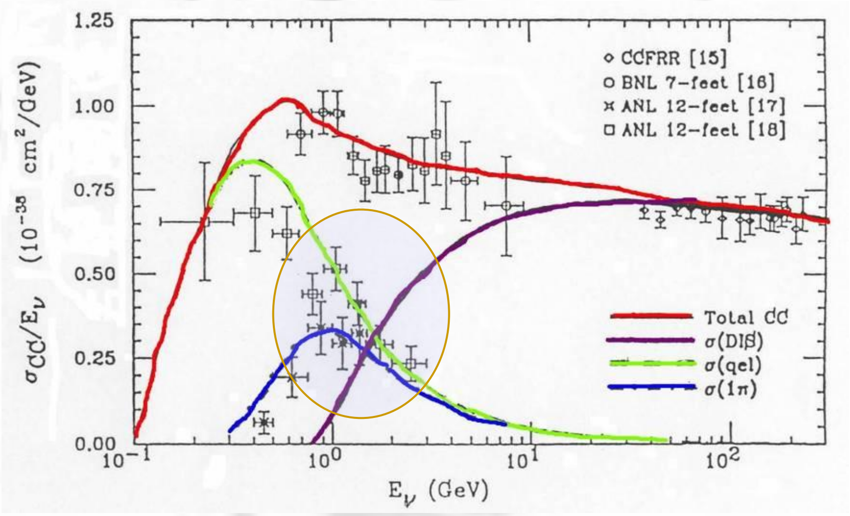




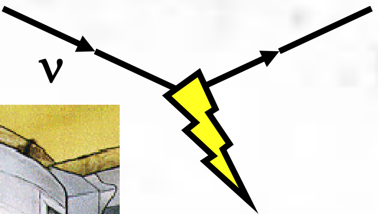
Threshold Effects

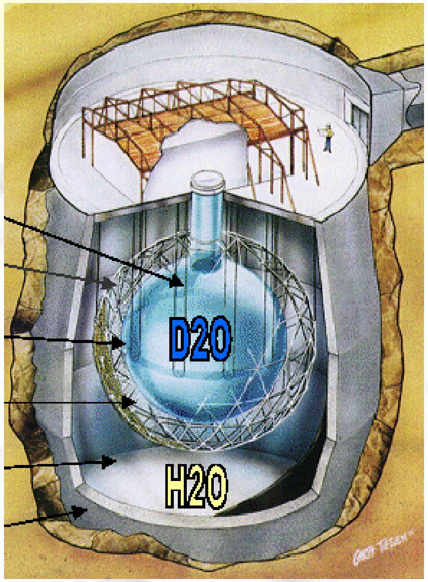
- At 1-few GeV, cross-section makes a transition between DIS-like and resonant/elastic
  - Why? "Binding energy" of target (nucleon) is ~1 GeV, comparable to mean Q^2



- What are other thresholds?
  - Binding energy of nucleus is >>(M_n-M_p)≈1 MeV, typically 1/10ths – 10s of MeV
  - Binding energies of atoms are <~$Z^2 m_e c^2 \alpha_{EM}/2$~10-$10^5$ eV
  - Binding energies of ν, ℓ , quarks (*into hypothetical constituents that we haven't found yet*) are > 10 TeV

Example: SNO





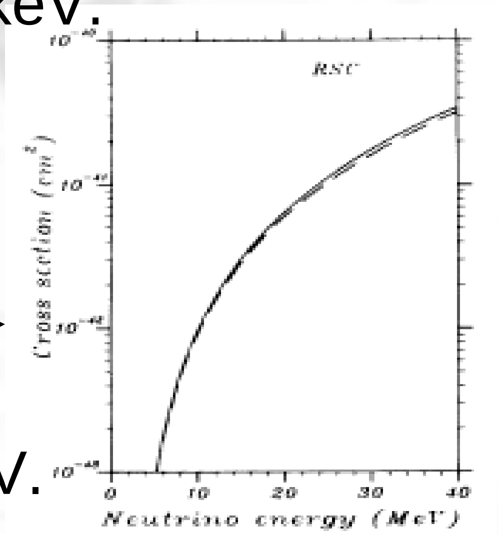
- Three reactions for observing ν from sun (E_ν ~ few MeV

ES $\nu_x + e^- \Rightarrow \nu_x + e^-$

  - ^{2}H, ^{16}O binding energies are 13.6eV, ~1 keV.
  - Therefore, e^- are "free". $\sigma \propto E_\nu$

CC $\nu_e + d \Rightarrow p + p + e^-$

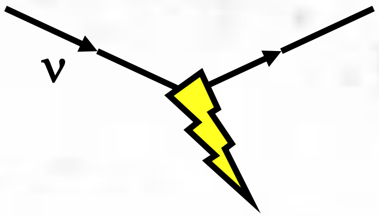
NC $\nu_x + d \Rightarrow p + n + \nu_x$



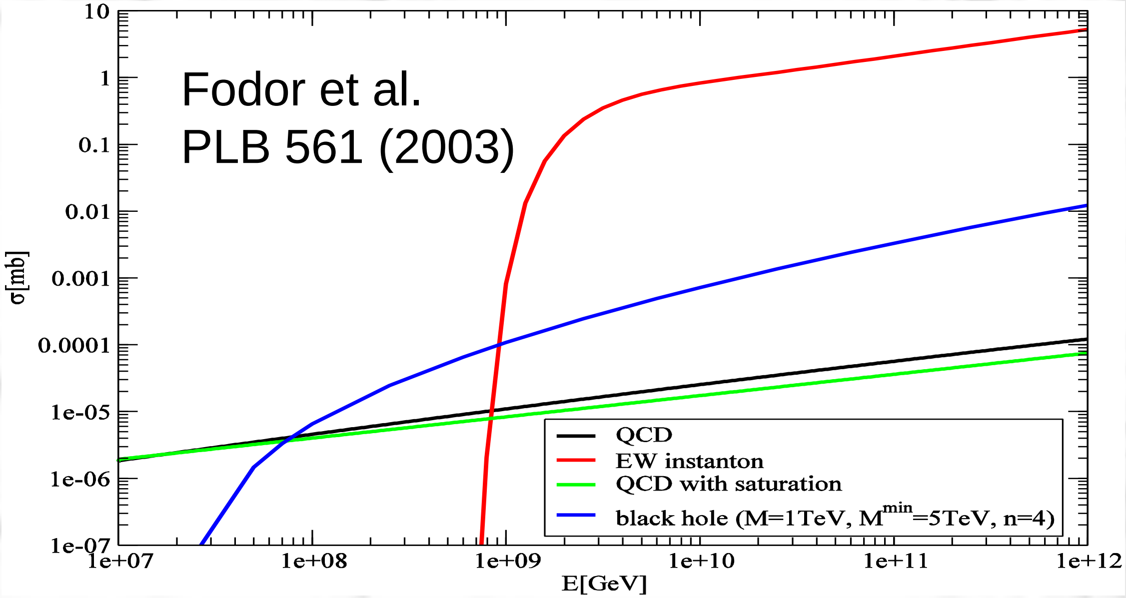
  - But binding energy of deuteron is 2.2 MeV. Energy threshold for NC of a few MeV.

(Bahcall, Kuboeara, Nozawa, PRD38 1030)

Example: Ultra-High Energies

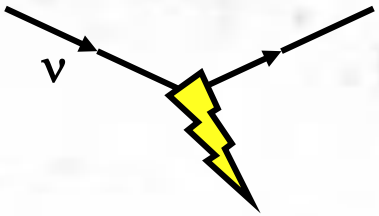


- At UHE, can we reach thresholds of non-SM processes?
  - E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Then no one knows what to expect…



Conclusions

What Should I Remember?



- Cross-sections have a simple physical interpretation
  - Weak interactions are different than electromagnetic or “geometric” cross-sections because of short range of force at low energies
- Weak interaction theory specifies neutrino interactions
  - High mass of W and Z bosons means scattering is independent of momentum exchanged
    - Neutrino scattering rate proportional to energy
  - Parity violation means couplings of two chiralities (right and left-handed fermions) are not identical
- Target (proton, nucleus) structure is a significant complication to theoretical prediction of cross-section
  - Particularly problematic near inelastic thresholds